RADIATION AND REFLECTION IN A RECTANGULAR CROSS-SECTION WAVEGUIDE: FINITE DIFFERENCE SIMULATION AND THEORETICAL FREQUENCY DOMAIN IMPEDANCE

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RADIATION AND REFLECTION IN A RECTANGULAR CROSS-SECTION WAVEGUIDE: FINITE DIFFERENCE SIMULATION AND THEORETICAL FREQUENCY DOMAIN IMPEDANCE

Dr. Jonathan Kemp
Dr. Stefan Bilbao
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The University of Edinburgh

University of St Andrews

Finite Difference Schemes

- Finite Difference Schemes provide a numerical method for Partial Differential Equations
- Spatial and Time Dimensions are discrete
- Here we will assume lossless wave propagation

Example: One Dimensional Wave Equation

- 1D Finite Difference (FD) scheme
- m is spatial sample number, (running from 1 to M)
- Evaluate from 2 < m < M - 1

$$p_m(n) = p_{m+1}(n-1) + p_{m-1}(n-1) - p_m(n-2)$$

- where n is the time sample number
- This implies a travelling wave speed of one spatial sample per time step (stable)

End Conditions

- Simulation with "ideal" clarinet end conditions
- Closed end at the left hand side means that perfect reflection from m=1, so $p_{m=0}(n) = p_{m=2}(n)$
- This gives closed end condition of:

$$p_{m=1}(n) = 2p_{m=2}(n-1) - p_{m=1}(n-2),$$

- "Ideal" open end at right hand side, $m = M$:

$$p_{m=M}(n) = 0.$$

Initialise to make Impulse

- Initialise the pressure vector to show impulse
- When impulse is at end it has forward and backward impulses coincident:
- $p(n=1) = [2, 0, 0, 0, 0, \ldots]$
- $p(n=2) = [0, 1, 0, 0, 0, \ldots]$
- then the 1D wave equation FD scheme will compute:
- $p(n=3) = [0, 0, 1, 0, 0, \ldots]$
- $p(n=4) = [0, 0, 0, 1, 0, \ldots]$ etc.

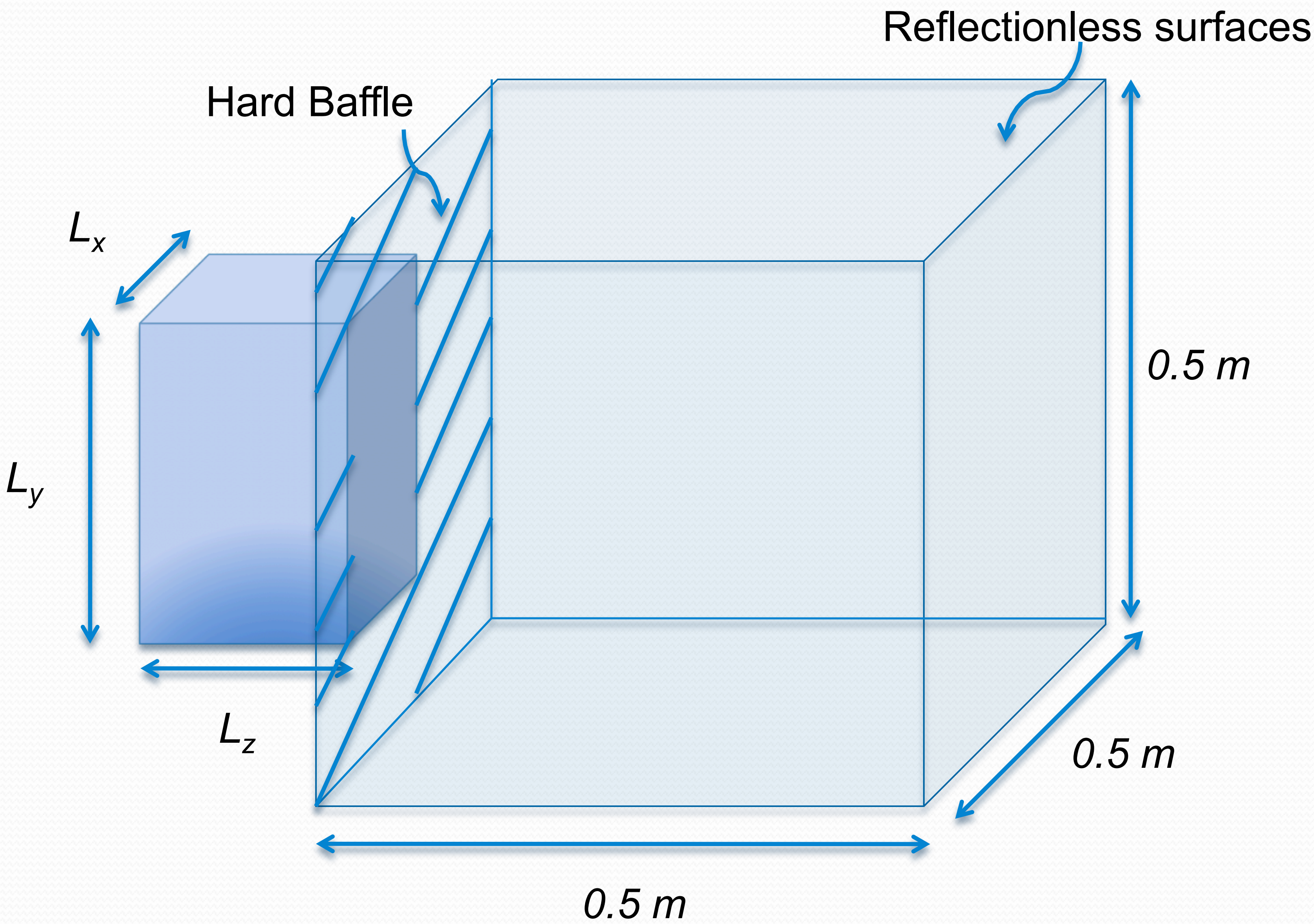
3D Tetrahedral Scheme

- Cubic grid of discrete spatial points
- "Tetrahedral" Scheme for the Finite Difference (FD) 3D Wave Equation
- See *Numerical Sound Synthesis*, S. Bilbao, Wiley, 2009
- Boundary conditions are complicated, but stable for a rectangular waveguide

Rectangular Waveguide

- F_s = 192 kHz
- L_x = 0.080 m
- L_y = 0.2365 m
- L_z = 0.1059 m
- Walls within the waveguide are perfectly reflecting (including the "input" surface perpendicular to z)
- Opening out into a baffle and reflection-less cube of wall length 0.5 m

Rectangular Waveguide



3D Tetrahedral Scheme

- Sound travels c/F_s in one time step where c is the speed of sound and F_s is the sample rate
- The distance between axial planes is $(\sqrt{3})c/F_s$
- Sound travels less than the distance between axial planes in one time sample
- Ideal impulse not possible: numerical dispersion

Backward Pulse

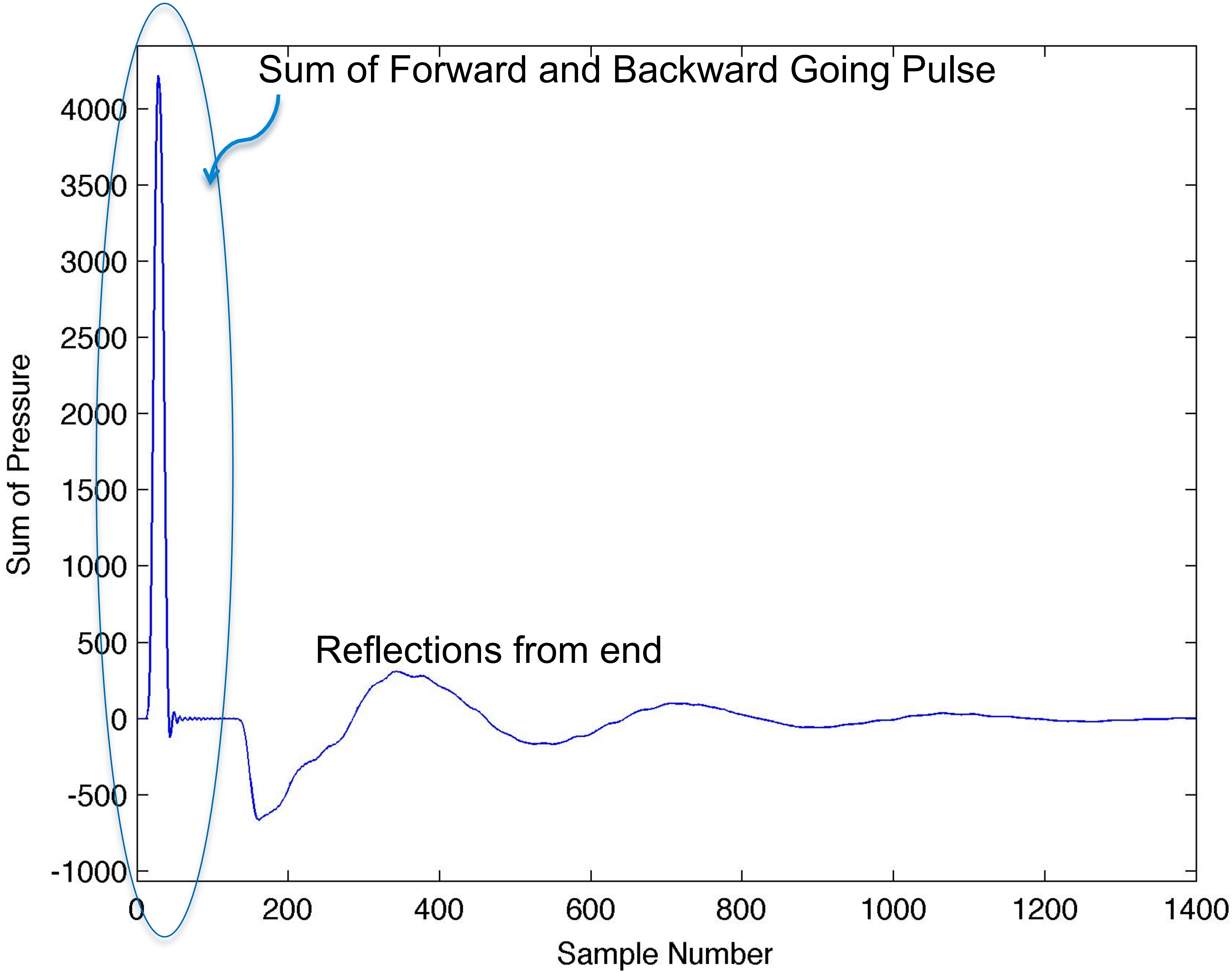
- A Hamming low pass filter function was created
- This was used as a function of z as initial condition within the waveguide
- This function was then moved by c/F_s to the negative z direction to make the next time step in initialisation
- This produced a backward going pulse shape

Input Impedance

- How can we check results of this time domain simulation with frequency domain theory?
- Use frequency domain radiation impedance of rectangular duct in infinite baffle known
- Project to input plane to get frequency domain input impedance (see my thesis)
- We need to calculate input impedance using information from FD simulation to compare

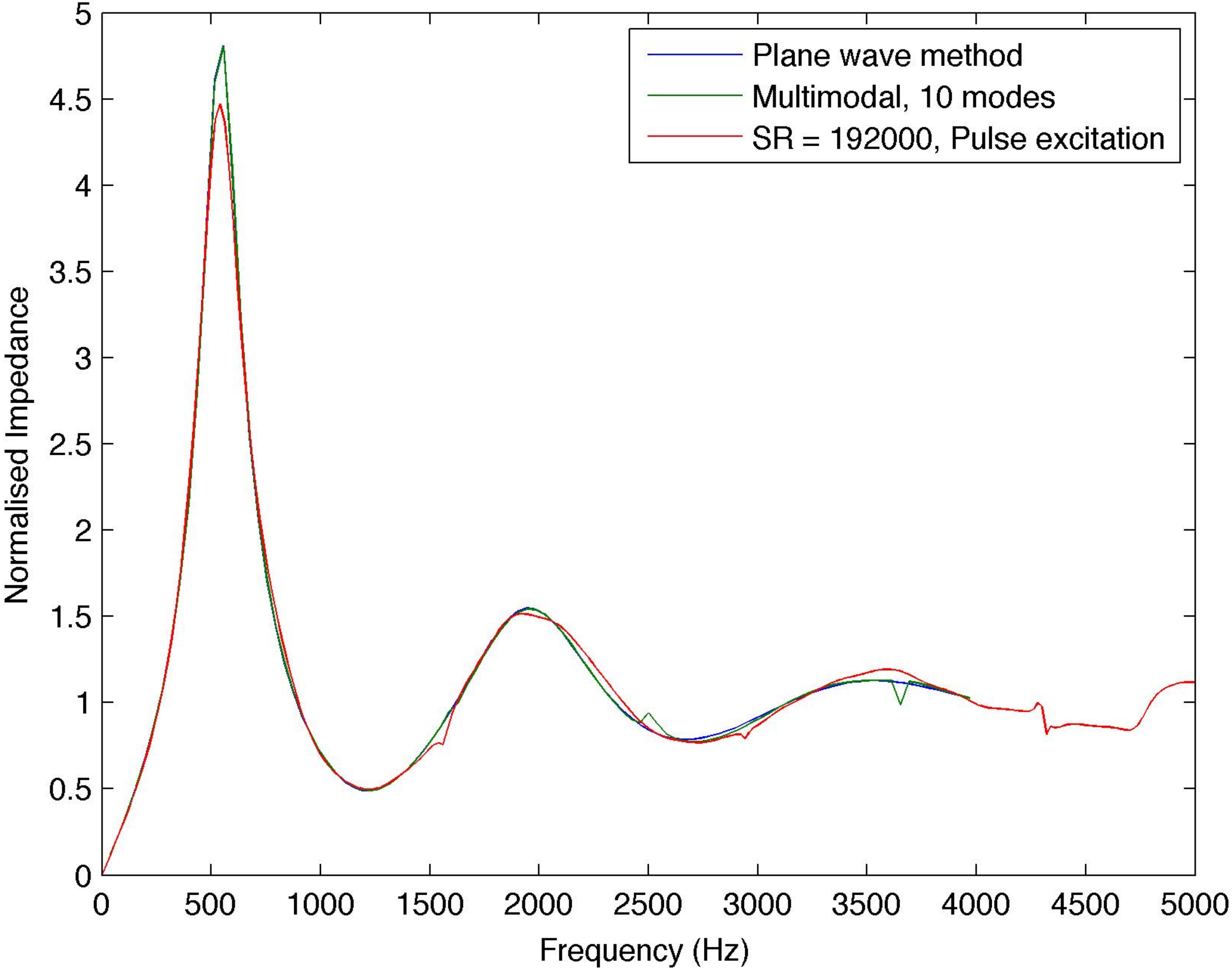
Input Impedance

- Sum of pressures on all points on the "input" surface recorded as a function of time in FD model
- This consists of double the input pulse (forward and backward going) followed by reflections from the end
- Input impedance can be defined as the FFT of the impulse response from a perfectly reflecting source
- We need to get the measurement of the forward going pulse without backward going pulse


Sum of Forward and Backward Going Pulse
Reflections from end
Sum of Pressure
Sample Number
4000
3500
3000
2500
2000
1500
1000
500
0
-500
-1000
0
200
400
600
800
1000
1200
1400

Input Impedance

- This initial pulse is halved
- The initial pulse is then windowed to get the “input pulse”
- This is then deconvolved from the response using frequency domain division
- Result, shown in the frequency domain, is the input impedance (as the input surface is perfectly reflecting)
- Agreement with theory is good



Plane wave method
Multimodal, 10 modes
SR = 192000, Pulse excitation
Normalised Impedance
Frequency (Hz)
0
0.5
1
1.5
2
2.5
3
3.5
4
4.5
5
0
500
1000
1500
2000
2500
3000
3500
4000
4500
5000

Narrower Waveguide

- Narrower waveguide will have much more energy retention within the waveguide:
- L_x = 0.040 m
- L_y = 0.040 m
- L_z = 0.1059 m
- Pulse is low pass filtered so gets wider each time it hits the end, until it looks like a standing wave

Any Questions?